الحساب التكاملي

$f \longrightarrow F$

تذكير:

$a \leq b$

$$\boxed{\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)}$$

$(kf)' = k \cdot f'$

$\boxed{\int_a^b k f(x)\,dx = k \int_a^b f(x)\,dx}$

$\boxed{\frac{f'}{f} \longrightarrow \ln|f|}$

$(3x+1)' = 3$

1 $I = \int_0^1 \frac{4}{3x+1}\,dx$

$= \frac{4}{3} \int_0^1 \frac{3}{3x+1}\,dx$

$= \frac{4}{3} \int_0^1 \frac{(3x+1)'}{3x+1}\,dx$

$= \frac{4}{3} [\ln|3x+1|]_0^1$

$= \frac{4}{3} [(\ln 4) - (0)]$

$= \frac{4}{3} \ln 4 = \ln 4^{\frac{4}{3}} = \boxed{\ln \sqrt[3]{4^4}}$

$\boxed{r \ln x = \ln x^r}$

$x^{\frac{n}{m}} = \sqrt[m]{x^n}$

2 $J = \int_0^1 \frac{1}{e^x+1}\,dx$

$\int_0^1 \frac{e^x}{(e^x+1)'}\,dx$

$(e^x+1)' = e^x$

$\frac{f'}{f} \longrightarrow \ln|f|$

$= \int_0^1 \frac{1+e^x - e^x}{e^x+1}\,dx$

$= \int_0^1 \cancel{\frac{1+e^x}{e^x+1}} - \frac{e^x}{e^x+1}\,dx$

$\boxed{\begin{array}{l} a \longrightarrow ax \\ 1 \longrightarrow x \end{array}}$

$= \int_0^1 1 - \frac{e^x}{e^x+1}\,dx$

$= \int_0^1 x - \frac{(e^x+1)'}{e^x+1}\,dx$

$= [x - \ln|e^x+1|]_0^1$

$e^0 = 1$

$= (1 - \ln(e+1)) - (0 - \ln(e^0+1))$

$= 1 - \ln(e+1) + \ln 2$

$= 1 + \ln\left(\frac{2}{e+1}\right)$

$\boxed{\begin{array}{l} \ln(a) - \ln(b) \\ = \ln\left(\frac{a}{b}\right) \end{array}}$

$\boxed{\ln e = 1}$

$= \ln e + \ln\left(\frac{2}{e+1}\right)$

$\boxed{\frac{f'}{f^2} \longrightarrow \ln|f|}$

$\boxed{\ln(a) + \ln(b) = \ln(a \times b)}$

$= \ln\left(\frac{2e}{e+1}\right)$

3 $K = \int_0^{\ln 2} e^{\frac{x}{2}}\,dx$

$\boxed{\begin{array}{l} e^x \longrightarrow e^x \\ e^{ax} \longrightarrow \frac{1}{a} e^{ax} \\ f' e^f \longrightarrow e^f \end{array}}$

$\frac{x}{2} = \frac{1}{2} \cdot x$

$= \int_0^{\ln 2} e^{\frac{1}{2}x}\,dx$

$= \left[\frac{1}{\frac{1}{2}} e^{\frac{1}{2}x}\right]_0^{\ln 2}$

$\boxed{\frac{\frac{a}{b}}{c} = \frac{a c}{b}}$

$= \left[2 e^{\frac{1}{2}x}\right]_0^{\ln 2}$

$= (2 e^{\frac{1}{2}\ln 2}) - (2 e^0)$

$= 2 e^{\ln 2^{\frac{1}{2}}} - 2$

$\boxed{\begin{array}{l} e^{a \ln x} \\ = e^{\ln x^a} = x^a \end{array}}$

$= 2 \cdot 2^{\frac{1}{2}} - 2$

$\boxed{x^{\frac{1}{2}} = \sqrt{x}}$

$= 2 \cdot \sqrt{2} - 2 = 2(\sqrt{2} - 1)$

$\boxed{4}$ $M = \int_0^1 e^{-x}(e^{3x}+1)\,dx$

$\boxed{e^x \cdot e^y = e^{x+y}}$

$\boxed{e^{-x} \longrightarrow -e^{-x}}$

$\boxed{e^{ax} \longrightarrow \frac{1}{a}e^{ax}}$

$\boxed{e^{-1} = \frac{1}{e}}$

$= \int_0^1 e^{-x} \cdot e^{3x} + e^{-x}\,dx$

$= \int_0^1 e^{2x} + e^{-x}\,dx$

$= \left[\frac{1}{2}e^{2x} - e^{-x}\right]_0^1$

$= \left(\frac{1}{2}e^2 - e^{-1}\right) - \left(\frac{1}{2}e^0 - e^0\right)$

$= \frac{1}{2}e^2 - \frac{1}{e} - \left(\frac{1}{2} - 1\right)$

$= \frac{1}{2}e^2 - \frac{1}{e} + \frac{1}{2}$

$= \frac{e^3 - 2 + e}{2e}$

$\boxed{5}$ $N = \int_{-1}^{1} |e^x - 1|\,dx$

علاقة شال

$|x| \to x \geq 0 ; |x| = x$

$|x| \to x \leq 0 ; |x| = -x$

$\boxed{\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx}$

Rm $|=0$

$e^x - 1 \geq 0$ ندرس

$\iff e^x \geq 1$

$\iff x \geq 0$

x	-1	0	1
e^x-1	$-$	0	$+$
$\lvert e^x-1 \rvert$	$-e^x+1$		e^x-1

$\int_{-1}^{1} |e^x - 1|\,dx = \int_{-1}^{0} -e^x + 1\,dx + \int_0^1 e^x - 1\,dx$

$= \left[-e^x + x\right]_{-1}^{0} + \left[e^x - x\right]_0^1$

$= \left[(-e^0 + 0) - (-e^{-1} - 1)\right] + \left[(e - 1) - (e^0 - 0)\right]$

$= -\cancel{1} + e^{-1} + \cancel{1} + e - 1 - 1$

$= e + \frac{1}{e} - 2 = \frac{e^2 + 1 - 2e}{e} = \frac{(e-1)^2}{e}$