# تصحيح الإمتحان الوطني
# 2024

## التمرين 1 :

1) أ) تحقق أن: $\forall n\in\mathbb{N}: \ u_{n+1}=4-\frac{6}{1+u_n}$

$$u_{n+1}=\frac{4u_n-2}{1+u_n}$$
$$=\frac{4u_n+4-4-2}{1+u_n}$$
$$=\frac{4(u_n+1)-6}{1+u_n}$$
$$=\frac{4(1+u_n)}{1+u_n}-\frac{6}{1+u_n}$$
$$=4-\frac{6}{1+u_n}.$$

ب) بين أن: $\forall n\in\mathbb{N}: \ 2\leq u_n\leq 4$

من أجل $n=0$ لدينا $2\leq u_0=4\leq 4$

ليكن $n\in\mathbb{N}$ نفترض أن $2\leq u_n\leq 4$

ونبين أن $2\leq u_{n+1}\leq 4$.

لدينا:
$$2\leq u_n\leq 4$$
$$\Rightarrow 3\leq 1+u_n\leq 5$$
$$\Rightarrow \frac{1}{5}\leq\frac{1}{1+u_n}\leq\frac{1}{3}$$
$$\Rightarrow -\frac{6}{3}\leq-\frac{6}{1+u_n}\leq-\frac{6}{5}$$
$$\Rightarrow -2\leq-\frac{6}{1+u_n}\leq-\frac{6}{5}$$
$$\Rightarrow 4-2\leq 4-\frac{6}{1+u_n}\leq 4-\frac{6}{5}$$
$$\Rightarrow 2\leq u_{n+1}\leq\frac{20-6}{5}$$
$$\Rightarrow 2\leq u_{n+1}\leq\frac{14}{5}\leq 4$$

إذا
$$\Rightarrow 2\leq u_{n+1}\leq 4$$

ومنه حسب مبدأ الترجع:
$$\forall n\in\mathbb{N}: \ 2\leq u_n\leq 4$$

2) أ) بين أن: $\forall n\in\mathbb{N}: \ u_{n+1}-u_n=\frac{(u_n-1)(2-u_n)}{1+u_n}$

$$u_{n+1}-u_n=\frac{4u_n-2}{1+u_n}-u_n$$
$$=\frac{4u_n-2-u_n(1+u_n)}{1+u_n}$$
$$=\frac{4u_n-2-u_n-u_n^2}{1+u_n}$$
$$=\boxed{\frac{-u_n^2+3u_n-2}{1+u_n}}.$$

$$\frac{(u_n-1)(2-u_n)}{1+u_n}=\frac{2u_n-u_n^2-2+u_n}{1+u_n}$$
$$=\boxed{\frac{-u_n^2+3u_n-2}{1+u_n}}$$

و/ك $u_{n+1}-u_n=\frac{(u_n-1)(2-u_n)}{1+u_n}$

ب) لنبين أن $u_n$ تناقصية.

لدينا:
$$u_{n+1}-u_n=\frac{(u_n-1)(2-u_n)}{1+u_n}$$
$$2\leq u_n\leq 4 \Rightarrow \boxed{1\leq u_n-1\leq 3}$$
$$2\leq u_n\leq 4 \Rightarrow -4\leq -u_n\leq -2$$
$$\Rightarrow \boxed{-2\leq 2-u_n\leq 0}$$
$$2\leq u_n\leq 4 \Rightarrow \boxed{3\leq 1+u_n\leq 5}$$

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$V_0 = \frac{2-U_0}{1-U_0} = \frac{2-4}{1-4} = \frac{-2}{-3} = \frac{2}{3}$. ج/

* $V_n = \frac{2-U_n}{1-U_n}$

$\Rightarrow V_n(1-U_n) = 2-U_n$

$\Rightarrow V_n - V_n \cdot U_n = 2-U_n$.

$\Rightarrow U_n - V_n \cdot U_n = 2-V_n$

$\Rightarrow U_n(1-V_n) = 2-V_n$

$\Rightarrow U_n = \frac{2-V_n}{1-V_n}$

$\Rightarrow U_n = \frac{1-V_n}{1-V_n} + \frac{1}{1-V_n}$

$\Rightarrow U_n = 1 + \frac{1}{1-V_n}$

$\Rightarrow \boxed{U_n = 1 + \frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}}$

$\lim_{n\to+\infty} U_n$.

$\lim_{+\infty} U_n = \lim_{+\infty} 1 + \frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}$

$= 1+1 = 2$

ج/ $\lim_{+\infty} \left(\frac{2}{3}\right)^{n+1} = 0 \quad , \; -1<\frac{2}{3}<1$

Pr. EDDAMI YASSINE

متقاربة:

لدينا $U_n$ تناقصية ومصغورة بـ 2

اذن $U_n$ متقاربة:

③ أ) لنبين أن $V_n$ هندسية أساسها $\frac{2}{3}$.

$V_{n+1} = \frac{2-U_{n+1}}{1-U_{n+1}}$

$= \frac{2 - \frac{4U_n-2}{1+U_n}}{1 - \frac{4U_n-2}{1+U_n}}$

$= \frac{\frac{2(1+U_n)-(4U_n-2)}{1+U_n}}{\frac{1+U_n-(4U_n-2)}{1+U_n}}$

$= \frac{2+2U_n-4U_n+2}{1+U_n-4U_n+2}$

$= \frac{-2U_n+4}{-3U_n+3}$ ②

$= \frac{2(-U_n+2)}{3(-U_n+1)}$

$= \frac{2}{3}\,\frac{2-U_n}{1-U_n}$

$= \frac{2}{3}\,V_n$.

اذن $V_n$ متتالية هندسية أساسها $q=\frac{2}{3}$.

ب) بين أن:

$\forall n\in\mathbb{N},\; U_n = 1+\frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}$.

لدينا $V_n$ هندسية اذن:

$V_n = V_p \cdot q^{n-p}$

$V_n = V_0 \cdot q^n = \frac{2}{3}\cdot\left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^{n+1}$

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$V_0 = \frac{2-U_0}{1-U_0} = \frac{2-4}{1-4} = \frac{-2}{-3} = \frac{2}{3}$. ج/

$*$

$$V_n = \frac{2-U_n}{1-U_n}$$

$\Rightarrow V_n(1-U_n) = 2-U_n$

$\Rightarrow V_n - V_n \cdot U_n = 2 - U_n$.

$\Rightarrow U_n - V_n \cdot U_n = 2 - V_n$

$\Rightarrow U_n(1-V_n) = 2 - V_n$

$\Rightarrow U_n = \frac{2-V_n}{1-V_n}$

$\Rightarrow U_n = \frac{1-V_n}{1-V_n} + \frac{1}{1-V_n}$

$\Rightarrow U_n = 1 + \frac{1}{1-V_n}$

$\Rightarrow \boxed{U_n = 1 + \frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}}$

ⓒ $\lim_{n \to +\infty} U_n$.

$$\lim_{+\infty} U_n = \lim_{+\infty} 1 + \frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}$$

$$= 1 + 1 = 2$$

ج/ $\lim_{+\infty} \left(\frac{2}{3}\right)^{n+1} = 0$ , $-1 < \frac{2}{3} < 1$

Pr. EDDAMI YASSINE

متقاربة:

لدينا $U_n$ تناقصية ومصغورة بـ 2

اذن $U_n$ متقاربة:

③ أ) لنبين أن $V_n$ هندسية أساسها $\frac{2}{3}$.

$$V_{n+1} = \frac{2-U_{n+1}}{1-U_{n+1}}$$

$$= \frac{2 - \frac{4U_n - 2}{1+U_n}}{1 - \frac{4U_n-2}{1+U_n}}$$

$$= \frac{\frac{2(1+U_n)-(4U_n-2)}{1+U_n}}{\frac{1+U_n-(4U_n-2)}{1+U_n}}$$

$$= \frac{2+2U_n-4U_n+2}{1+U_n-4U_n+2}$$

$$= \frac{-2U_n+4}{-3U_n+3}$$

② 

$$= \frac{2(-U_n+2)}{3(-U_n+1)}$$

$$= \frac{2}{3} \cdot \frac{2-U_n}{1-U_n}$$

$$= \frac{2}{3} V_n.$$

اذن $V_n$ متتالية هندسية أساسها $q = \frac{2}{3}$.

ب) بين أن:

$$\forall n \in \mathbb{N}, \ U_n = 1 + \frac{1}{1-\left(\frac{2}{3}\right)^{n+1}}.$$

لدينا $V_n$ هندسية اذن:

$$V_n = V_p \cdot q^{n-p}$$

$$V_n = V_0 \cdot q^n = \frac{2}{3} \cdot \left(\frac{2}{3}\right)^n = \left(\frac{2}{3}\right)^{n+1}$$

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التمرين 2 :

1) بين أن $(P) : 2x-2y+z+3=0.$

المستوى $(P)$ مار من A و $\vec{n}$ منظمية عليه اذن:

$(P) : 2x-2y+z+d=0.$

وبما أن $A(-1;0;-1)\in(P)$

اذن:

$$2x_A-2y_A+z_A+d=0$$

$$2\times(-1)-2\times0+(-1)+d=0$$

$$-2-0-1+d=0$$

$$-3+d=0$$

$$\boxed{d=3}$$

اذن $\boxed{(P) : 2x-2y+z+3=0}$

2) المعادلة الديكارتية للفلكة $(S)$:

$\Omega(2;-1;0)$ و $R=5.$

$$(x-2)^2+(y-(-1))^2+(z-0)^2=5^2$$

$$(x-2)^2+(y+1)^2+z^2=25$$

$$x^2-4x+4+y^2+2y+1+z^2=25$$

$$x^2+y^2+z^2-4x+2y+5-25=0$$

$$\boxed{x^2+y^2+z^2-4x+2y-20=0}$$

3) أ)

$$d(\Omega;(P))=\frac{|2x_\Omega-2y_\Omega+z_\Omega+3|}{\sqrt{2^2+(-2)^2+1^2}}$$

$$=\frac{|2\times2-2\times(-1)+0+3|}{\sqrt{4+4+1}}$$

$$=\frac{|4+2+3|}{\sqrt{9}}=\frac{9}{3}=\boxed{3}$$

ب) لدينا $d(\Omega;(P))=3<R$

اذن المستوى $(P)$ يقطع الفلكة $(S)$ وفق دائرة $(\Gamma)$ مركزها:

$$r=\sqrt{R^2-d^2}$$

$$=\sqrt{5^2-3^2}$$

$$=\sqrt{25-9}$$

$$=\sqrt{16}$$

$$\boxed{r=4}$$

4) أ) لدينا $(\Delta)$ المستقيم المار من $\Omega$ وعمودي على المستوى $(P)$ اذن $\vec{n}$ متجهة موجهة للمستقيم $(\Delta)$:

$$(\Delta) : \begin{cases} x=x_\Omega+at \\ y=y_\Omega+bt \\ z=z_\Omega+ct \end{cases} \quad / t\in\mathbb{R}$$

3

$$(\Delta) : \begin{cases} x=2+2t \\ y=-1-2t \\ z=t \end{cases} \quad / t\in\mathbb{R}$$

ب) لدينا $(\Delta)$ مار من $\Omega$ وعمودي على $(P)$ والمستوى $(P)$ يقطع الفلكة $(S)$ وفق دائرة $(\Gamma)$ اذن الاسقاط العمودي ل $\Omega$ على $(P)$ هو H

$$H=(P)\cap(\Delta).$$

$$\begin{cases} 2x-2y+z+3=0 \\ \begin{cases} x=2+2t \\ y=-1-2t \\ z=t \end{cases} / t\in\mathbb{R} \end{cases}$$

$$\Rightarrow 2(2+2t)-2(-1-2t)+t+3=0$$

$$\Rightarrow 4+4t+2+4t+t+3=0$$

$$\Rightarrow 9t+9=0$$

$$\Rightarrow 9t=-9 \Rightarrow \boxed{t=-1}$$

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التمرين 2 :

1) بين أن $(P): 2x-2y+z+3=0.$

المستوى $(P)$ مار من $A$ و $\vec{n}$ منظمية عليه اذن:

$(P): 2x-2y+z+d=0.$

وبما أن $A(-1;0;-1)\in(P)$

اذن: $2x_A-2y_A+z_A+d=0$

$2\times(-1)-2\times0+(-1)+d=0$

$-2-0-1+d=0$

$-3+d=0$

$\boxed{d=3}$

اذن $\boxed{(P): 2x-2y+z+3=0}$

2) المعادلة الديكارتية للفلكة $(S)$:

$\Omega(2;-1;0)$ و $R=5.$

$(x-2)^2+(y-(-1))^2+(z-0)^2=5^2$

$(x-2)^2+(y+1)^2+z^2=25$

$x^2-4x+4+y^2+2y+1+z^2=25$

$x^2+y^2+z^2-4x+2y+5-25=0$

$\boxed{x^2+y^2+z^2-4x+2y-20=0}$

3) أ)

$$d(\Omega;(P))=\frac{|2x_\Omega-2y_\Omega+z_\Omega+3|}{\sqrt{2^2+(-2)^2+1^2}}$$

$$=\frac{|2\times2-2\times(-1)+0+3|}{\sqrt{4+4+1}}$$

$$=\frac{|4+2+3|}{\sqrt{9}}=\frac{9}{3}=\boxed{3}$$

ب) لدينا $d(\Omega;(P))=3<R$

اذن المستوى $(P)$ يقطع الفلكة $(S)$ وفق دائرة $(\Gamma)$ مركزها:

$r=\sqrt{R^2-d^2}$

$=\sqrt{5^2-3^2}$

$=\sqrt{25-9}$

$=\sqrt{16}$

$\boxed{r=4}$

4) أ) لدينا $(\Delta)$ المستقيم المار من $\Omega$ وعمودي على المستوى $(P)$ اذن $\vec{n}$ متجهة موجهة للمستقيم $(\Delta)$:

$$(\Delta):\begin{cases}x=x_\Omega+at\\ y=y_\Omega+bt\\ z=z_\Omega+ct\end{cases}\quad / t\in\mathbb{R}$$

$$(\Delta):\begin{cases}x=2+2t\\ y=-1-2t\\ z=t\end{cases}\quad / t\in\mathbb{R}$$

ب) لدينا $(\Delta)$ مار من $\Omega$ وعمودي على $(P)$ والمستوى $(P)$ يقطع الفلكة $(S)$ وفق دائرة $(\Gamma)$ اذن المسقط العمودي ل $\Omega$ على $(P)$ هو $H$

$H=(P)\cap(\Delta).$

$$\begin{cases}2x-2y+z+3=0\\ \begin{cases}x=2+2t\\ y=-1-2t\\ z=t\end{cases} / t\in\mathbb{R}\end{cases}$$

$\Rightarrow 2(2+2t)-2(-1-2t)+t+3=0$

$\Rightarrow 4+4t+2+4t+t+3=0$

$\Rightarrow 9t+9=0$

$\Rightarrow 9t=-9 \Rightarrow \boxed{t=-1}$

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$$\begin{cases} x_H = 2 + 2\times(-1) = 0 \\ y_H = -1 - 2\times(-1) = 1 \\ z_H = -1 \end{cases}$$

$$\boxed{H(0\,;1\,;-1).}$$

ج) لنبين أن $(\Delta)$ واسط القطعة $[AB]$.

$\vec{n}(2\,;-2\,;1) \qquad \vec{AB}(2\,;2\,;0)$

$$\begin{aligned} \vec{n}\cdot\vec{AB} &= 2\times 2 + (-2)\times 2 + 1\times 0 \\ &= 4-4 \\ &= 0 \end{aligned}$$

إذن $(AB) \perp (\Delta)$.

لنحدد احداثيات منتصف القطعة $[AB]$

$\dfrac{x_A+x_B}{2} = \dfrac{1+(-1)}{2} = 0$

$\dfrac{y_A+y_B}{2} = \dfrac{2+0}{2} = \dfrac{2}{2} = 1.$

$\dfrac{z_A+z_B}{2} = \dfrac{(-1)+(-1)}{2} = \dfrac{-2}{2} = -1.$

إذن $H$ منتصف القطعة $[AB]$.

و $H \in (\Delta)$

إذن $(\Delta)$ هو واسط القطعة $[AB]$

## التمرين 3:

1)

$$a = \sqrt{3}(1-i)$$

$$\begin{aligned} |a| &= |\sqrt{3}(1-i)| \\ &= |\sqrt{3}|.|1-i| \\ &= \sqrt{3}.\sqrt{1^2+(-1)^2} \\ &= \sqrt{3}.\sqrt{2} \\ &= \sqrt{6} \end{aligned}$$

$$\begin{aligned} a &= \sqrt{3}(1-i) \\ &= \sqrt{6}\left(\frac{\sqrt{2}}{2} - i\,\frac{\sqrt{2}}{2}\right) \\ &= \sqrt{6}\left(\cos\left(-\frac{\pi}{4}\right) + i.\sin\left(-\frac{\pi}{4}\right)\right) \end{aligned}$$

$$\arg(a) \equiv -\frac{\pi}{4} \ [2\pi].$$

1) 2)

$$\begin{aligned} \frac{b}{a} &= \frac{2+\sqrt{3}+i}{\sqrt{3}(1-i)} \\ &= \frac{2+\sqrt{3}+i}{\sqrt{3}-\sqrt{3}i} \\ &= \frac{(2+\sqrt{3}+i)(\sqrt{3}+\sqrt{3}i)}{(\sqrt{3}-\sqrt{3}i)(\sqrt{3}+\sqrt{3}i)} \\ &= \frac{(2+\sqrt{3}).\sqrt{3} + (2+\sqrt{3}).\sqrt{3}i + i\sqrt{3} + i^2.\sqrt{3}}{\sqrt{3}^2 - (i\sqrt{3})^2} \\ &= \frac{2\sqrt{3} + 3 + 2\sqrt{3}i + 3i + i\sqrt{3} - \sqrt{3}}{3+3} \\ &= \frac{3+\sqrt{3} + 3\sqrt{3}i + 3i}{6} \\ &= \frac{3+\sqrt{3}}{6} + \frac{3(\sqrt{3}+1)i}{6} \end{aligned}$$

4

$$\frac{b}{a} = \frac{3+\sqrt{3}}{6} + \frac{\sqrt{3}+1}{2}i$$

$$\begin{aligned} \frac{b}{a} &= \frac{3+\sqrt{3}}{6} + \frac{\sqrt{3}+1}{2}i \\ &= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2} + \frac{\sqrt{3}+1}{2}\times\frac{3}{3+\sqrt{3}}\,i\right) \\ &= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2} + \frac{\cancel{\sqrt{3}+1}}{2}\times\frac{3}{\sqrt{3}(\cancel{1+\sqrt{3}})}\,i\right) \\ &= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right) \\ &= \frac{3+\sqrt{3}}{3}.\left(\cos\frac{\pi}{3} + i.\sin\frac{\pi}{3}\right) \\ &= \frac{3+\sqrt{3}}{3}.e^{i.\frac{\pi}{3}}. \end{aligned}$$

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$$\arg(a) \equiv -\frac{\pi}{4}\ [2\pi].$$

2

$$\frac{b}{a} = \frac{2+\sqrt{3}+i}{\sqrt{3}(1-i)}$$

$$= \frac{2+\sqrt{3}+i}{\sqrt{3}-\sqrt{3}i}$$

$$= \frac{(2+\sqrt{3}+i)(\sqrt{3}+\sqrt{3}i)}{(\sqrt{3}-\sqrt{3}i)(\sqrt{3}+\sqrt{3}i)}$$

$$= \frac{(2+\sqrt{3})\cdot\sqrt{3}+(2+\sqrt{3})\cdot\sqrt{3}i+i\sqrt{3}+i^2\cdot\sqrt{3}}{\sqrt{3}^2-(i\sqrt{3})^2}$$

$$= \frac{2\sqrt{3}+3+2\sqrt{3}i+3i+i\sqrt{3}-\sqrt{3}}{3+3}$$

$$= \frac{3+\sqrt{3}+3\sqrt{3}i+3i}{6}$$

4

$$= \frac{3+\sqrt{3}}{6}+\frac{3(\sqrt{3}+1)i}{6}$$

$$\frac{b}{a} = \frac{3+\sqrt{3}}{6}+\frac{\sqrt{3}+1}{2}i$$

$$\frac{b}{a} = \frac{3+\sqrt{3}}{6}+\frac{\sqrt{3}+1}{2}i$$

$$= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2}+\frac{\sqrt{3}+1}{2}\times\frac{3}{3+\sqrt{3}}i\right)$$

$$= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2}+\frac{\cancel{\sqrt{3}+1}}{2}\times\frac{3}{\sqrt{3}(\cancel{1+\sqrt{3}})}i\right)$$

$$= \frac{3+\sqrt{3}}{3}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$= \frac{3+\sqrt{3}}{3}\cdot\left(\cos\frac{\pi}{3}+i\cdot\sin\frac{\pi}{3}\right)$$

$$= \frac{3+\sqrt{3}}{3}\cdot e^{i\cdot\frac{\pi}{3}}.$$

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$$\begin{cases} x_H = 2+2\times(-1) = 0 \\ y_H = -1-2\times(-1) = 1 \\ z_H = -1 \end{cases}$$

$$\boxed{H(0\,;1\,;-1).}$$

ج) لنبين أن $(\Delta)$ واسط القطعة $[AB]$.

$\vec{n}(2;-2;1)$ $\quad \vec{AB}(2;2;0)$

$$\vec{n}\cdot\vec{AB} = 2\times2+(-2)\times2+1\times0$$
$$= 4-4$$
$$= 0$$

اذن $(AB) \perp (\Delta)$.

لنحدد احداثيات منتصف القطعة $[AB]$

$$\frac{x_A+x_B}{2} = \frac{1+(-1)}{2} = 0$$

$$\frac{y_A+y_B}{2} = \frac{2+0}{2} = \frac{2}{2} = 1.$$

$$\frac{z_A+z_B}{2} = \frac{(-1)+(-1)}{2} = \frac{-2}{2} = -1.$$

اذن $H$ منتصف القطعة $[AB]$.

و $H \in (\Delta)$

اذن $(\Delta)$ هو واسط القطعة $[AB]$

## التمرين 3:

1)

$$a = \sqrt{3}(1-i)$$

$$|a| = |\sqrt{3}(1-i)|$$
$$= |\sqrt{3}|.|1-i|$$
$$= \sqrt{3}.\sqrt{1^2+(-1)^2}$$
$$= \sqrt{3}.\sqrt{2}$$
$$= \sqrt{6}$$

$$a = \sqrt{3}(1-i)$$
$$= \sqrt{6}\left(\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)$$
$$= \sqrt{6}\left(\cos\left(-\frac{\pi}{4}\right)+i.\sin\left(-\frac{\pi}{4}\right)\right)$$

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3) i)

$$z' - w = e^{i\theta}(z - w)$$

$$z' - 0 = e^{i\pi/6} \cdot (z - 0)$$

$$\Rightarrow z' = e^{i\pi/6} \cdot z$$

$$\Rightarrow z' = \left(\cos\frac{\pi}{6} + i \cdot \sin\frac{\pi}{6}\right) \cdot z$$

$$\Rightarrow z' = \left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) \cdot z$$

$$\Rightarrow \boxed{z' = \frac{1}{2}(\sqrt{3} + i) \cdot z}$$

*

$$R(A) = A'$$

$$\Rightarrow a' = \frac{1}{2}(\sqrt{3} + i) \cdot a.$$

$$\arg(a') \equiv \arg\left(\frac{1}{2}(\sqrt{3}+i) \cdot a\right) \; [2\pi]$$

$$\equiv \arg\left(\frac{1}{2}(\sqrt{3}+i)\right) + \arg(a) \; [2\pi]$$

$$\equiv \frac{\pi}{6} - \frac{\pi}{4} \; [2\pi]$$

$$\equiv \frac{2\pi - 3\pi}{12} \; [2\pi]$$

$$\boxed{\arg(a') \equiv -\frac{\pi}{12} \; [2\pi]}$$

c)

$$R(A') = A''$$

$$\Rightarrow a'' = \frac{1}{2}(\sqrt{3} + i) \cdot a'$$

$$\Rightarrow a'' = \left(\frac{1}{2}(\sqrt{3}+i)\right)^2 \cdot a.$$

$$|a''| = \left|\frac{\sqrt{3}}{2} + \frac{1}{2}i\right|^2 \cdot |a|$$

$$= 1 \cdot \sqrt{6}$$

$$= \boxed{\sqrt{6}}.$$

$$\arg(a'') \equiv \arg\left(\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2 \cdot a\right) \; [2\pi]$$

$$\equiv \arg\left(\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2\right) + \arg(a) \; [2\pi]$$

$$\equiv 2 \cdot \arg\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) + \arg(a) \; [2\pi]$$

2) ب) لدينا

$$\frac{b}{a} = \frac{3+\sqrt{3}}{3} \cdot e^{i\frac{\pi}{3}}$$

$$\Rightarrow b = \left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right) \cdot a.$$

$$|b| = \left|\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a\right|$$

$$= \left|\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right| \cdot |a|$$

$$= \frac{3+\sqrt{3}}{3} \cdot \sqrt{6}.$$

$$= \frac{(3+\sqrt{3}) \cdot \sqrt{2}}{\sqrt{3}} = \boxed{\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}}} = \boxed{\sqrt{2} + \sqrt{6}}$$

$$\arg(b) \equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a\right) \; [2\pi]$$

$$\equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right) + \arg(a) \; [2\pi]$$

$$\equiv \frac{\pi}{3} - \frac{\pi}{4} \; [2\pi]$$

$$\arg(b) \equiv \frac{\pi}{12} \; [2\pi]$$

$$b = \frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)$$

$$= (\sqrt{2} + \sqrt{6})\left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right) = (\sqrt{2} + \sqrt{6})e^{i\frac{\pi}{12}}$$

$$b^{24} = \left(\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)\right)^{24}$$

$$= \left(\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}}\right)^{24} \cdot \left(\cos\left(24\frac{\pi}{12}\right) + i \cdot \sin\left(24\frac{\pi}{12}\right)\right)$$

$$= \left(\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}}\right)^{24} \cdot \left(\underbrace{\cos(2\pi)}_{1} + i \cdot \underbrace{\sin(2\pi)}_{0}\right)$$

$$b^{24} = \left(\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}}\right)^{24}$$

$$= \left(\frac{3\sqrt{6} + 3\sqrt{2}}{3}\right)^{24} = (\sqrt{6} + \sqrt{2})^{24} \in \mathbb{R}$$

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3) i)

$z' - w = e^{i\theta}(z - w)$

$z' - 0 = e^{i\pi/6} \cdot (z - 0)$

$\Rightarrow z' = e^{i\pi/6} \cdot z$

$\Rightarrow z' = \left(\cos\frac{\pi}{6} + i \cdot \sin\frac{\pi}{6}\right) \cdot z$

$\Rightarrow z' = \left(\frac{\sqrt{3}}{2} + i\,\frac{1}{2}\right) \cdot z$

$\Rightarrow \boxed{z' = \frac{1}{2}(\sqrt{3} + i) \cdot z}$

*

$R(A) = A'$

$\Rightarrow a' = \frac{1}{2}(\sqrt{3}+i) \cdot a.$

$\arg(a') \equiv \arg\left(\frac{1}{2}(\sqrt{3}+i) \cdot a\right) \; [2\pi]$

$\equiv \arg\left(\frac{1}{2}(\sqrt{3}+i)\right) + \arg(a) \; [2\pi]$

$\equiv \frac{\pi}{6} - \frac{\pi}{4} \; [2\pi]$

$\equiv \frac{2\pi - 3\pi}{12} \; [2\pi]$

$\boxed{\arg(a') \equiv -\frac{\pi}{12} \; [2\pi]}$

ج)

$R(A') = A''$

$\Rightarrow a'' = \frac{1}{2}(\sqrt{3}+i) \cdot a'$

$\Rightarrow a'' = \left(\frac{1}{2}(\sqrt{3}+i)\right)^2 \cdot a.$

$|a''| = \left|\frac{\sqrt{3}}{2} + \frac{1}{2}i\right|^2 \cdot |a|$

$= 1 \cdot \sqrt{6}$

$= \boxed{\sqrt{6}}.$

$\arg(a'') \equiv \arg\left(\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2 \cdot a\right) \; [2\pi]$

$\equiv \arg\left(\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^2\right) + \arg(a) \; [2\pi]$

$\equiv 2 \cdot \arg\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) + \arg(a) \; [2\pi]$

2) ب) لدينا $\frac{b}{a} = \frac{3+\sqrt{3}}{3} \cdot e^{i\frac{\pi}{3}}$

$\Rightarrow b = \left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right) \cdot a.$

$|b| = \left|\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a\right|$

$= \left|\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right| \cdot |a|$

$= \frac{3+\sqrt{3}}{3} \cdot \sqrt{6}.$

$= \frac{(3+\sqrt{3}) \cdot \sqrt{2}}{\sqrt{3}} = \boxed{\frac{3\sqrt{2} + \sqrt{6}}{\sqrt{3}}} = \boxed{\sqrt{2}+\sqrt{6}}$

$\arg(b) \equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a\right) \; [2\pi]$

$\equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3}\right) + \arg(a) \; [2\pi]$

$\equiv \frac{\pi}{3} - \frac{\pi}{4} \; [2\pi]$

$\arg(b) \equiv \frac{\pi}{12} \; [2\pi]$

$b = \frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)$

$= (\sqrt{2}+\sqrt{6})\left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right) = (\sqrt{2}+\sqrt{6})e^{i\frac{\pi}{12}}$

$b^{24} = \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)\right)^{24}$

$= \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}}\right)^{24} \cdot \left(\cos\left(24\frac{\pi}{12}\right) + i \cdot \sin\left(24\frac{\pi}{12}\right)\right)$

$= \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}}\right)^{24} \cdot \left(\underbrace{\cos(2\pi)}_{1} + i \cdot \underbrace{\sin(2\pi)}_{0}\right)$

$b^{24} = \left(\frac{3\sqrt{2}+\sqrt{6}}{\sqrt{3}}\right)^{24}$

$= \left(\frac{3\sqrt{6}+3\sqrt{2}}{3}\right)^{24} = (\sqrt{6}+\sqrt{2})^{24} \in \mathbb{R}$

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$$\boxed{b' = \frac{3+\sqrt{3}}{3} \cdot \overline{a}}.$$

7)

$$\arg\left(\frac{b'-0}{a-0}\right) \equiv \arg\left(\frac{b'}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{\frac{3+\sqrt{3}}{3} \cdot \overline{a}}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{\frac{3+\sqrt{3}}{3} \cdot i \cdot a}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot i\right) [2\pi]$$

لأن :

$$\equiv \arg(i) [2\pi] \quad \left(\frac{3+\sqrt{3}}{3} \in \mathbb{R}^+\right)$$

$$\equiv \frac{\pi}{2} [2\pi]$$

اذن المثلث $OAB'$ قائم الزاوية في $O$.

$$\arg(a'') \equiv \frac{2\pi}{6} + \left(-\frac{\pi}{4}\right) [2\pi]$$

$$\equiv \frac{\pi}{3} - \frac{\pi}{4} [2\pi]$$

$$\equiv \boxed{\frac{\pi}{12} [2\pi]}$$

$$a'' = \sqrt{6} \cdot \left(\cos\frac{\pi}{12} + i \cdot \sin\frac{\pi}{12}\right)$$

$$\boxed{a'' = \sqrt{6} \cdot e^{i \cdot \pi/12}}$$

6)

$$\frac{b-0}{a''-0} = \frac{(\sqrt{2}+\sqrt{6}) \cdot (\cos\pi/12 + i \cdot \sin\pi/12)}{\sqrt{6} \cdot e^{i\pi/12}}$$

$$= \frac{(\sqrt{2}+\sqrt{6}) \cdot \cancel{e^{i\pi/12}}}{\sqrt{6} \cdot \cancel{e^{i\pi/12}}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{12}+6}{6}$$

$$= \frac{\sqrt{3}+3}{3} \in \mathbb{R}$$

اذن $O$ ، $A''$ و $B$ نقط مستقيمية.

## التمرين 4 :

| |
|---|
| ① ① ② ③ |
| ① ① ② |

1)

①① أو ②②

$$P(A) = \frac{C_4^2 + C_2^2}{C_7^2} = \frac{6+1}{21} = \frac{7}{21} = \frac{1}{3}.$$

2)

①③ أو ②②

$$P(B) = \frac{C_4^1 \cdot C_1^1 + C_2^2}{C_7^2} = \frac{4\times1+1}{21} = \frac{5}{21}$$

3)

$A \cap B =$ ②②

$$P(A\cap B) = \frac{C_2^2}{C_7^2} = \frac{1}{21}.$$

4)

$$P(A)\times P(B) = \frac{1}{3}\times\frac{5}{21} = \frac{5}{63} \neq \frac{1}{21} = P(A\cap B)$$

اذن الحدثان $A$ و $B$ غير مستقلين.

لأن $P(A)\times P(B) \neq P(A\cap B)$

2)

1) $R(B) = B'$

$$\Rightarrow b' = \frac{1}{2}(\sqrt{3}+i) \cdot b$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \cdot \frac{3+\sqrt{3}}{3} \cdot e^{i\pi/3} \cdot a$$

$$b' = \frac{3+\sqrt{3}}{3} \cdot \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) \cdot \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \cdot a$$

$$= \frac{3+\sqrt{3}}{3}\left(\cancel{\frac{\sqrt{3}}{4}} + i\frac{3}{4} + \frac{1}{4}i - \cancel{\frac{\sqrt{3}}{4}}\right) \cdot a$$

$$= \frac{3+\sqrt{3}}{3} \cdot i \cdot a$$

$$= \frac{3+\sqrt{3}}{3} i\left(\sqrt{3}(1-i)\right)$$

$$= \frac{3+\sqrt{3}}{3}\sqrt{3}\left(i(1-i)\right)$$

$$= \frac{3+\sqrt{3}}{3} \cdot \sqrt{3} \cdot (i+1)$$

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$$\boxed{b' = \frac{3+\sqrt{3}}{3} \cdot \bar{a}}$$

$$\arg\left(\frac{b'-0}{a-0}\right) \equiv \arg\left(\frac{b'}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{\frac{3+\sqrt{3}}{3} \cdot \bar{a}}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{\frac{3+\sqrt{3}}{3} \cdot i \cdot a}{a}\right) [2\pi]$$

$$\equiv \arg\left(\frac{3+\sqrt{3}}{3} \cdot i\right) [2\pi]$$

لأن:

$$\equiv \arg(i) [2\pi] \quad \left(\frac{3+\sqrt{3}}{3} \in \mathbb{R}^+\right)$$

$$\equiv \frac{\pi}{2} [2\pi]$$

اذن المثلث $OAB'$ قائم الزاوية في $O$.

التمرين 4:

| | | | |
|---|---|---|---|
| 1 1 | | 2 | 3 |
| 1 1 | | 2 | |

1) 11 أو 22

$$P(A) = \frac{C_4^2 + C_2^2}{C_7^2} = \frac{6+1}{21} = \frac{7}{21} = \frac{1}{3}.$$

2) 13 أو 22

$$P(B) = \frac{C_4^1 \cdot C_1^1 + C_2^2}{C_7^2} = \frac{4\times1+1}{21} = \frac{5}{21}$$

3) $A\cap B = 22$

$$P(A\cap B) = \frac{C_2^2}{C_7^2} = \frac{1}{21}.$$

4) $$P(A)\times P(B) = \frac{1}{3}\times\frac{5}{21} = \frac{5}{63} \neq \frac{1}{21} = P(A\cap B)$$

اذن الحدثان $A$ و $B$ غير مستقلان.

لأن $P(A)\times P(B) \neq P(A\cap B)$

Pr: EDDAMI YASSINE

$$\arg(a'') \equiv \frac{2\pi}{6} + \left(-\frac{\pi}{4}\right) [2\pi]$$

$$\equiv \frac{\pi}{3} - \frac{\pi}{4} [2\pi]$$

$$\equiv \boxed{\frac{\pi}{12} [2\pi]}$$

$$a'' = \sqrt{6}\cdot\left(\cos\frac{\pi}{12} + i\cdot\sin\frac{\pi}{12}\right)$$

$$\boxed{a'' = \sqrt{6}\cdot e^{i\cdot\pi/12}}$$

6)

$$\frac{b-0}{a''-0} = \frac{(\sqrt{2}+\sqrt{6})\cdot\left(\cos\pi/12 + i\cdot\sin\pi/12\right)}{\sqrt{6}\cdot e^{i\pi/12}}$$

$$= \frac{(\sqrt{2}+\sqrt{6})\cdot \cancel{e^{i\pi/12}}}{\sqrt{6}\cdot\cancel{e^{i\pi/12}}}$$

$$= \frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{12}+6}{6}$$

$$= \frac{\sqrt{3}+3}{3} \in \mathbb{R}$$

اذن $O$, $A''$ و $B$ نقط مستقيمية.

7) 1) $R(B) = B'$

$$\Rightarrow b' = \frac{1}{2}(\sqrt{3}+i)\cdot b$$

$$= \left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)\cdot\frac{3+\sqrt{3}}{3}\cdot e^{i\pi/3}\cdot a$$

$$b' = \frac{3+\sqrt{3}}{3}\cdot\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)\cdot\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\cdot a$$

$$= \frac{3+\sqrt{3}}{3}\left(\cancel{\frac{\sqrt{3}}{4}} + i\frac{3}{4} + \frac{1}{4}i - \cancel{\frac{\sqrt{3}}{4}}\right)\cdot a$$

$$= \frac{3+\sqrt{3}}{3}\cdot i\cdot a$$

$$= \frac{3+\sqrt{3}}{3} i\left(\sqrt{3}(1-i)\right)$$

$$= \frac{3+\sqrt{3}}{3}\sqrt{3}\left(i(1-i)\right)$$

$$= \frac{3+\sqrt{3}}{3}\cdot\sqrt{3}\cdot(i+1)$$

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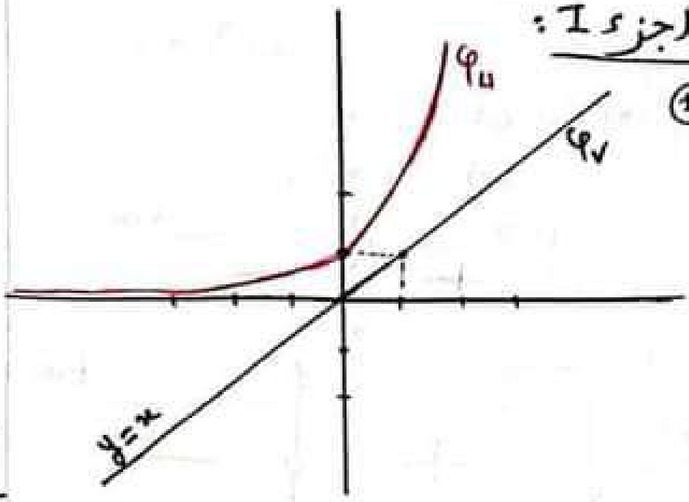
المسألة :

الجزء I :

1)




2) لدينا $(\varphi_v)$ تحت $(\varphi_u)$ في الشكل السابق

اذن : $u(x) > v(x)$.

ومنه $u(x) - v(x) > 0$

$e^x - x > 0$.

7

3)

$$\mathcal{A} = \int_0^1 |u(x) - v(x)| \,.\, dx \,.\, u.a.$$

$$= \int_0^1 |e^x - x| \,.\, dx \,.\, u.a$$

$$= \int_0^1 (e^x - x) \,.\, dx \,.\, u.a \qquad \text{لأن } (e^x - x > 0)$$

$$= \left[e^x - \frac{x^2}{2}\right]_0^1 . \, u.a.$$

$$= \left[e - \frac{1}{2} - (e^0 - 0)\right] . \, u.a.$$

$$= \left(e - \frac{1}{2} - 1\right) u.a.$$

$$= e - \frac{3}{2} \; . u.a.$$

الجزء II :

$f(x) = x + 1 - \ln(e^x - x)$.

1) أ)

$D_f = \{x \in \mathbb{R} \,/\, e^x - x > 0\}$.

$= \mathbb{R}$ حسب السؤال 2) I .

ج)

$$f(x) = x + 1 - \ln(e^x - x)$$

$$= x + 1 - \ln\left(e^x(1 - x.e^{-x})\right)$$

$$= x + 1 - \ln(e^x) - \ln(1 - x.e^{-x})$$

$$= \cancel{x} + 1 - \cancel{x} - \ln(1 - x.e^{-x})$$

$$= \boxed{1 - \ln(1 - x.e^{-x})}.$$

د)

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} 1 - \ln(1 - x.e^{-x})$$

$$= \lim_{+\infty} 1 - \ln\left(1 - \frac{x}{e^x}\right)$$

$$= \lim_{+\infty} 1 - \ln\left(1 - \frac{1}{\frac{e^x}{x}}\right) \qquad \frac{e^x}{x} \to +\infty$$

$$= 1 - \ln(1)$$

$$= 1 \qquad \left(\text{car } \lim_{+\infty} \frac{e^x}{x} = +\infty\right)$$

$(C_f)$ يقبل مقارب أفقي معادلته $y = 1$ بجوار $+\infty$.

2) أ)

$$\lim_{-\infty} f(x) = \lim_{-\infty} x + 1 - \ln(e^x - x)$$

$$= -\infty$$

لأن :

$\lim_{-\infty} x + 1 = -\infty$

$\lim_{-\infty} e^x - x = +\infty$

$\lim_{-\infty} \ln(e^x - x) = +\infty$.

ب)

$$f(x) = x + 1 - \ln(e^x - x) \qquad \text{لأن: } x < 0 \Rightarrow -x > 0$$

$$= x + 1 - \ln\left(-x\left(\frac{e^x}{-x} + 1\right)\right)$$

$$= x + 1 - \ln(-x) - \ln\left(1 + \frac{e^x}{-x}\right)$$

$$\boxed{f(x) = x + 1 - \ln(-x) - \ln\left(1 - \frac{1}{x e^{-x}}\right)}$$

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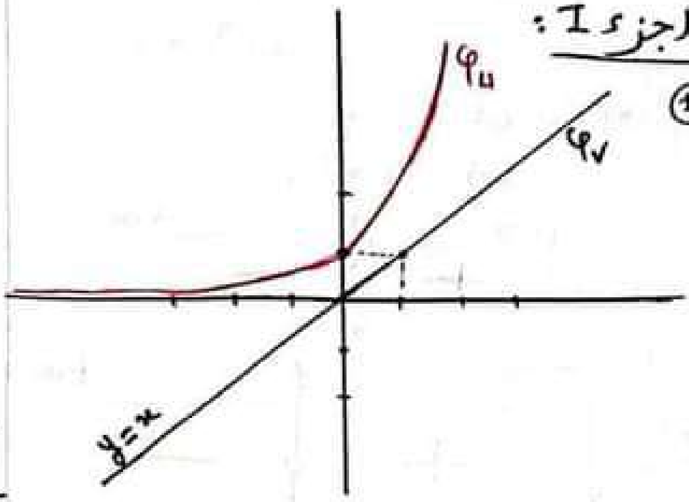
المسألة :

الجزء I :

① 


② لدينا $(\varphi_v)$ تحت $(\varphi_u)$ في الشكل السابق

إذن : $u(x) > v(x)$.

ومنه $u(x) - v(x) > 0$

$e^x - x > 0$.

7

③

$$\mathcal{A} = \int_0^1 |u(x) - v(x)| . dx . u.a.$$

$$= \int_0^1 |e^x - x| . dx . u.a$$

$$= \int_0^1 (e^x - x) . dx . u.a \quad \left(\text{c'à } e^x - x > 0\right)$$

$$= \left[e^x - \frac{x^2}{2}\right]_0^1 . u.a.$$

$$= \left[e - \frac{1}{2} - (e^0 - 0)\right] . u.a.$$

$$= \left(e - \frac{1}{2} - 1\right) . u.a.$$

$$= e - \frac{3}{2} \; . u.a.$$

الجزء II :

$f(x) = x + 1 - \ln(e^x - x)$.

①أ

$D_f = \{x \in \mathbb{R} / e^x - x > 0\}$.

$= \mathbb{R}$ . حسب السؤال ②I

ب

$$f(x) = x + 1 - \ln(e^x - x)$$

$$= x + 1 - \ln\left(e^x(1 - x.e^{-x})\right)$$

$$= x + 1 - \ln(e^x) - \ln(1 - x.e^{-x})$$

$$= \cancel{x} + 1 - \cancel{x} - \ln(1 - x.e^{-x})$$

$$= \boxed{1 - \ln(1 - x.e^{-x})}.$$

ج

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} 1 - \ln(1 - x.e^{-x})$$

$$= \lim_{+\infty} 1 - \ln\left(1 - \frac{x}{e^x}\right)$$

$$= \lim_{+\infty} 1 - \ln\left(1 - \frac{1}{\frac{e^x}{x}}\right) \quad \frac{e^x}{x} \to +\infty$$

$$= 1 - \ln(1)$$

$$= 1 \quad \left(\text{car } \lim_{+\infty} \frac{e^x}{x} = +\infty\right)$$

$(\varphi_f)$ يقبل مقارب أفقي معادلته $y = 1$ بجوار $+\infty$.

②أ

$$\lim_{-\infty} f(x) = \lim_{-\infty} x + 1 - \ln(e^x - x)$$

$$= -\infty$$

c'à :

$$\lim_{-\infty} x + 1 = -\infty$$

$$\lim_{-\infty} e^x - x = +\infty$$

$$\lim_{-\infty} \ln(e^x - x) = +\infty.$$

ب

$$f(x) = x + 1 - \ln(e^x - x) \quad \left(\text{c'à } x < 0 \Rightarrow -x > 0\right)$$

$$= x + 1 - \ln\left(-x\left(\frac{e^x}{-x} + 1\right)\right)$$

$$= x + 1 - \ln(-x) - \ln\left(1 + \frac{e^x}{-x}\right)$$

$$\boxed{f(x) = x + 1 - \ln(-x) - \ln\left(1 - \frac{1}{xe^{-x}}\right)}$$

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ب) اشارة $f'$ هي اشارة $1-x$ لأن $e^x-x>0$.

$$f'(x)=0 \iff 1-x=0 \iff x=1.$$

| $x$ | $-\infty$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|
| $1-x$ | | $+$ | $0$ | $-$ | |

| $x$ | $-\infty$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|
| $f'(x)$ | | $+$ | $0$ | $-$ | |
| $f(x)$ | $-\infty$ | $\nearrow$ | $2-\ln(e-1)$ | $\searrow$ | $1$ |

$$f(1)=1+1-\ln(e^1-1)=2-\ln(e-1)$$

ج) لدينا $f$ متصلة على $\mathbb{R}$ (لأنها قابلة للاشتقاق على $\mathbb{R}$) وبالخصوص على $[-1;0]$.

و $f$ تزايدية قطعا على $[-1;0]$ (أنظر جدول التغيرات)

$$f(-1)=-1+1-\ln(e^{-1}+1)$$
$$=-\ln\left(\frac{1}{e}+1\right)$$

$$\frac{1}{e}+1>1 \Rightarrow \ln\left(\frac{1}{e}+1\right)>0$$
$$\Rightarrow \boxed{-\ln\left(\frac{1}{e}+1\right)<0}$$

$$f(0)=0+1-\ln(e^0-0)$$
$$=1-\ln(1)$$
$$=1>0$$

اذن $f(-1)\times f(0)<0$

ومنه حسب مبرهنة القيم الوسيطية T.V.I المعادلة $f(x)=0$ تقبل حلا وحيدا $\alpha$ في المجال $]-1;0[$.

2) أ)

$$\lim_{-\infty}\frac{f(x)}{x}=\lim_{-\infty}\frac{x+1-\ln(-x)-\ln\left(1-\frac{1}{xe^{-x}}\right)}{x}$$

$$=\lim_{-\infty}\frac{x}{x}+\frac{1}{x}-\frac{\ln(-x)}{x}-\frac{\ln\left(1-\frac{1}{xe^{-x}}\right)}{x}$$

$$=\lim_{-\infty}1+\frac{1}{x}+\frac{\ln(-x)}{-x}-\frac{\ln\left(1+\frac{1}{-xe^{-x}}\right)}{x}$$

$$=\boxed{1}$$

لأن:

$$\lim_{-\infty}\frac{1}{x}=0\ ;\ \lim_{-\infty}\frac{\ln(-x)}{-x}=0$$

$$\lim_{-\infty}\frac{\ln\left(1+\frac{1}{-x.e^{-x}}\right)}{x}=0.$$

$$\lim_{-\infty}-x.e^{-x}=+\infty.$$

$$*\ \lim_{-\infty}f(x)-x=\lim_{-\infty}1-\ln(-x)-\ln\left(1-\frac{1}{xe^{-x}}\right)$$

$$=\lim_{-\infty}1-\underbrace{\ln(-x)}_{+\infty}-\underbrace{\ln\left(1+\frac{1}{-x.e^{-x}}\right)}_{0}$$

$$=-\infty$$

اذن $(C_f)$ يقبل فرعا شلجميا في اتجاه المستقيم الذي معادلته $y=x$ بجوار $-\infty$.

3) أ)

$$f'(x)=\left(x+1-\ln(e^x-x)\right)'$$

$$=1+0-\frac{(e^x-x)'}{e^x-x}$$

$$=1-\frac{e^x-1}{e^x-x}$$

$$=\frac{e^x-x-e^x+1}{e^x-x}$$

$$\boxed{f'(x)=\frac{1-x}{e^x-x}}$$

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ب) اشارة $f'$ هي اشارة $1-x$ لأن $e^x - x > 0$.

$$f'(x)=0 \Leftrightarrow 1-x=0 \Leftrightarrow x=1.$$

| $x$ | $-\infty$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|
| $1-x$ | | $+$ | $0$ | $-$ | |

| $x$ | $-\infty$ | | $1$ | | $+\infty$ |
|---|---|---|---|---|---|
| $f'(x)$ | | $+$ | $0$ | $-$ | |
| $f(x)$ | $-\infty$ | $\nearrow$ | $2-\ln(e-1)$ | $\searrow$ | $1$ |

$$f(1)=1+1-\ln(e^1-1)=2-\ln(e-1)$$

ج) لدينا $f$ متصلة على $\mathbb{R}$ (لأنها ق ش على $\mathbb{R}$) وبالخصوص على $[-1;0]$.

و $f$ تزايدية قطعا على $[-1;0]$ (أنظر جدول التغيرات)

$$f(-1)=-1+1-\ln(e^{-1}+1) = -\ln\left(\frac{1}{e}+1\right)$$

$$\frac{1}{e}+1>1 \Rightarrow \ln\left(\frac{1}{e}+1\right)>0 \Rightarrow \boxed{-\ln\left(\frac{1}{e}+1\right)<0}$$

$$f(0)=0+1-\ln(e^0-0) = 1-\ln(1) = 1>0$$

اذن $f(-1)\times f(0)<0$

ومنه حسب مبرهنة القيم الوسيطية T.V.I المعادلة $f(x)=0$ تقبل حلا وحيدا $\alpha$ في المجال $]-1;0[$.

ح)

$$\lim_{-\infty} \frac{f(x)}{x} = \lim_{-\infty} \frac{x+1-\ln(-x)-\ln\left(1-\frac{1}{xe^{-x}}\right)}{x}$$

$$=\lim_{-\infty} \frac{x}{x}+\frac{1}{x}-\frac{\ln(-x)}{x}-\frac{\ln\left(1-\frac{1}{xe^{-x}}\right)}{x}$$

$$=\lim_{-\infty} 1+\frac{1}{x}+\frac{\ln(-x)}{-x}-\frac{\ln\left(1+\frac{1}{-xe^{-x}}\right)}{x}$$

$$=\boxed{1}$$

لأن:

$$\lim_{-\infty}\frac{1}{x}=0 \ ; \ \lim_{-\infty}\frac{\ln(-x)}{-x}=0$$

$$\lim_{-\infty}\frac{\ln\left(1+\frac{1}{-x.e^{-x}}\right)}{x}=0.$$

$$\lim_{-\infty} -x.e^{-x}=+\infty.$$

$$* \lim_{-\infty} f(x)-x = \lim_{-\infty} 1-\ln(-x)-\ln\left(1-\frac{1}{xe^{-x}}\right)$$

$$=\lim_{-\infty} 1-\underbrace{\ln(-x)}_{+\infty}-\underbrace{\ln\left(1+\frac{1}{-x.e^{-x}}\right)}_{0}$$

$$=-\infty$$

اذن $(C_f)$ يقبل فرع شلجمي في اتجاه المستقيم الذي معادلته $y=x$ بجوار $-\infty$.

3) أ)

$$f'(x)=\left(x+1-\ln(e^x-x)\right)'$$

$$=1+0-\frac{(e^x-x)'}{e^x-x}$$

$$=1-\frac{e^x-1}{e^x-x}$$

$$=\frac{e^x-x-e^x+1}{e^x-x}$$

$$\boxed{f'(x)=\frac{1-x}{e^x-x}}$$

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4) أ) نلاحظ من خلال الشكل أن $(\mathcal{C}_f)$
يقطع المستقيم $y=x$ في نقطتين
أفصولهما $\alpha$ و $\beta$.

ب)

$$f(\alpha)=\alpha \Rightarrow f(\alpha)-\alpha=0$$

$$\Rightarrow 1-\ln(e^{\alpha}-\alpha)=0$$

$$\Rightarrow -\ln(e^{\alpha}-\alpha)=-1$$

$$\Rightarrow \ln(e^{\alpha}-\alpha)=1$$

$$\Rightarrow \boxed{e^{\alpha}-\alpha=e} \quad ①$$

$$f(\beta)=\beta \Rightarrow f(\beta)-\beta=0$$

$$\Rightarrow 1-\ln(e^{\beta}-\beta)=0$$

$$\Rightarrow \ln(e^{\beta}-\beta)=1.$$

$$\Rightarrow \boxed{e^{\beta}-\beta=e} \quad ②$$

$$① \text{ et } ② \Rightarrow e^{\alpha}-\alpha=e^{\beta}-\beta$$

$$\Rightarrow \boxed{e^{\alpha}-e^{\beta}=\alpha-\beta}.$$

5) أ) لدينا $g$ متصلة على $]-\infty;1]$
(لأنها قابلة للاشتقاق على $]-\infty;1]$)
و $g$ تزايدية قطعا على $]-\infty;1]$
(جدول التغيرات)
اذن: $g$ تقبل دالة عكسية $g^{-1}$ معرفة على
مجال $J$:

$$J=g(I)=g(]-\infty;1])$$

$$=]\lim_{-\infty} g(x);\ g(1)]$$

$$=]-\infty;2-\ln(e-1)].$$

ب) لدينا $f(0)=1$

اذن $g(0)=1.$

بما أن $g$ قابلة للاشتقاق في $0$
وأن $g'(0)=\dfrac{1-0}{e^0-0}=\dfrac{1}{1}=1 \neq 0$

اذن $g^{-1}$ قابلة للاشتقاق في $1$.

$$(g^{-1})'(1)=\frac{1}{g'(g^{-1}(1))}$$

$$=\frac{1}{g'(0)}$$

$$=\frac{1}{1}$$

$$(g^{-1})'(1)=1.$$

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Fin

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4) أ) نلاحظ من خلال الشكل أن $(C_f)$ يقطع المستقيم $y=x$ في نقطتين أفصولهما $\alpha$ و $\beta$.

ب)

$$f(\alpha)=\alpha \Rightarrow f(\alpha)-\alpha=0$$

$$\Rightarrow 1-\ln(e^\alpha-\alpha)=0$$

$$\Rightarrow -\ln(e^\alpha-\alpha)=-1$$

$$\Rightarrow \ln(e^\alpha-\alpha)=1$$

$$\Rightarrow \boxed{e^\alpha-\alpha=e} \quad (1)$$

$$f(\beta)=\beta \Rightarrow f(\beta)-\beta=0$$

$$\Rightarrow 1-\ln(e^\beta-\beta)=0$$

$$\Rightarrow \ln(e^\beta-\beta)=1.$$

$$\Rightarrow \boxed{e^\beta-\beta=e} \quad (2)$$

$$(1) \text{ et } (2) \Rightarrow e^\alpha-\alpha=e^\beta-\beta$$

$$\Rightarrow \boxed{e^\alpha-e^\beta=\alpha-\beta}.$$

5) أ) لدينا $g$ متصلة على $]-\infty;1]$
(لأنها قابلة للاشتقاق على $]-\infty;1]$)
و $g$ تزايدية قطعا على $]-\infty;1]$
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إذن: $g$ تقبل دالة عكسية $g^{-1}$ معرفة على مجال $J$:

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Fin

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